Dhw Check:
Box 9: Dividing Polynomials #5
Box 10: Dividing Polynomials #10

$$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$\frac{-14 \pm \sqrt{(14)^2-4(4)(49)}}{2(4)}$$

$$\frac{-14 \pm \sqrt{196-784}}{8}$$

$$\frac{-14 \pm \sqrt{-588}}{8}$$

$$\frac{-14 \pm \sqrt{-1 \cdot 196 \cdot 3}}{8}$$

$$\frac{-14 \pm 14i\sqrt{3}}{8}$$

$$\boxed{\frac{-7 \pm 7i\sqrt{3}}{4}}$$

Warm-up

1. Solve $16x^3 - 686 = 0$ by factoring.

$2(8x^3-343)=0$

$2x, 7$ $\quad 2(2x-7)(4x^2+14x+49)=0$

$2x-7=0 \qquad a=4, b=14, c=49$

$2x=7$

$x=\frac{7}{2}$

2. The width of a box is 2m less than the length. The height is 1m less than the length. The volume is 60m³. What is the width of the box?

$x-2$ - width

x - length

$x-1$ - height

$x(x-2)(x-1)=60$

$x(x-2)(x-1)-60=0$

Calculator $x=5$

$5-2=\boxed{3m}$

3. When $x^2 + 3x + b$ is divided by $x + a$, the quotient is $x + 6$ and the remainder is 14. Find a and b.

Notes Sec. 3.6 Roots of Polynomials

Rational Root Theorem: to find the possible rational roots of a polynomial function, find the reduced form of $\frac{p}{q}$ where p = factor of the constant and q = factor of the leading coefficient.

We are going to use the rational root theorem to find actual roots of any polynomial function. Then, you can use these roots, synthetic division, and the quadratic formula to find any remaining roots.

① list the factors of p
② list the factors of q
③ write as $\frac{p}{q}$
④ plug in the calculator

$\boxed{y=}$ $\boxed{2^{nd}}$ $\boxed{\text{window}}$ → $\boxed{\text{indep ask}}$

Problem 1: Using the Rational Root Theorem

a) What are the rational roots of $15x^3 - 32x^2 + 3x + 2 = 0$?

p: 2   1, 2

q: 15   1, 3, 5, 15

$\pm 1, \pm\frac{1}{3}, \pm\frac{1}{5}, \pm\frac{1}{15}, \pm 2, \pm\frac{2}{3}, \pm\frac{2}{5}, \pm\frac{2}{15}$

$\left\{\frac{1}{3}, -\frac{1}{5}, 2\right\}$

b) What are the rational roots of $2x^3 + x^2 - 7x - 6 = 0$?

p: 6   1, 2, 3, 6 $\pm 1, \pm 2, \pm 3, \pm 6, \pm \frac{1}{2}, \pm \frac{3}{2}$

q: 2   1, 2

$-1, 2, -\frac{3}{2}$

<u>Conjugate Root Theorem</u>: irrational roots occur in conjugate pairs. For example, if $a + bi$ is a root, then $a - bi$ is also a root. Likewise, if $a + \sqrt{b}$ is a root, then $a - \sqrt{b}$ is a root.

Problem 2: Identify Roots

a) A quartic (4) polynomial has rational coefficients. If $\sqrt{2}$ and $1 + i$ are roots of the polynomial, what are the other two roots?

Remember! The degree of the polynomial will tell you how many roots you should end up with!!!

$\sqrt{2}, 1 + i$

$-\sqrt{2}, 1 - i$

b) A cubic polynomial as roots $3-2i$ and $\frac{5}{2}$. What is the other root?

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$3+2i, 3-2i, \frac{5}{2}$

Problem 3: Using Conjugates to Construct a Polynomial

1. Find any additional roots using the conjugate root theorem
2. Write your equation in intercept (root) form
3. Multiply
4. Write your answer in standard form (there should be no imaginary numbers or square roots in your answer!)

a) What is a third-degree polynomial with roots -4 *and* $2i$?

$-4, 2i, -2i$

$x=-4 \quad x=2i \quad x=-2i$

$x+4=0 \quad x-2i=0 \quad x+2i=0$

$(x+4)(x-2i)(x+2i)=0$

$(x+4)(x^2+4)=0$

$x^3+4x+4x^2+16=0$

$x^3+4x^2+4x+16$

$(x-2i)(x+2i)$

$x(x+2i)-2i(x+2i)$

$x^2 \cancel{+2xi} \cancel{-2xi} -4i^2$

$(x^2+4) \quad (-4)(-1)$

3b) What is a ~~quartic~~ cubic polynomial with roots $2-3i, 8,$ ~~and 2~~?

4) Write a polynomial function with the given roots

A) $4+\sqrt{5}\ and\ \cancel{2}$ -5

B) [illegible] $- 7i$ *and* $2 - \sqrt{11}$